

Late time evolution of brane gas cosmology and compact internal dimensions

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We study the late-time behavior of a universe in the framework of brane gas cosmology. We investigate the evolution of a universe with a gas of supergravity particles and a gas of branes. Considering the case when different dimensions are anisotropically wrapped by various branes, we have derived Friedman-like equations governing the dynamics of wrapped and unwrapped subvolumes. We point out that the compact internal dimensions are wrapped by three or higher dimensional branes.

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1 Introduction

Unifying gravity with other forces of nature strongly suggests that there may be more than three spatial dimensions in the unifying scale. Some of these are hidden from the low energy observers. This is related to the cosmological question why we have only three spatial dimensions. There had been earlier attempts to answer this question [1, 2, 3, 4, 5]. One of the notable is the idea of string cosmology proposed by Brandenberger and Vafa (BV) [5]. This is a mechanism to generate dynamically the spatial dimensionality of spacetime and to explain the problem of initial singularity. The key ingredient of this model is based on the symmetry of string theory called T-duality. With this symmetry the spacetime has a topology of nine dimensional torus and its dynamics is driven by a gas of fundamental strings. Cosmology based on this string-modified Einstein-Friedman equations was studied in many directions [6, 7, 8, 9].

Developments in string theory during the last decade revealed that it is not a theory of only strings and has richer structure of branes [10]. Recently there have been attempts to understand the cosmological evolution in this new framework of string theory with D-branes [11, 12] and the mechanism of BV was considered in [13]. This model of brane gas cosmology was studied extensively [14, 15, 16, 17]. In the picture of brane gas cosmology, the universe starts from hot, dense gas of D-branes in thermal equilibrium. The winding modes of branes obstruct the growing of the spatial dimensions. Branes with opposite winding numbers can annihilate if their world volume interacts. Thus hierarchy of scales can be achieved between the wrapped and unwrapped dimensions. Of particular interest is whether the unwrapped configuration of branes can successfully inflate to make the unwrapped dimensions grow indefinitely while making the wrapped ones remain small or at least grow much slowly [16, 17].

In [16], the authors studied the late-time cosmology in M-theory with a supergravity particle gas and wrapped 2-brane gas. In this paper, we study the late-time behavior of brane gas cosmology in the context of string theory by extending the formalism of [16] to general p -brane gas. We investigate the behavior of a universe with a gas of supergravity particle and a gas of wrapped p -brane. Considering the case where different directions are anisotropically wrapped by various branes, we will argue the possible hierarchical evolution of scales between wrapped and unwrapped dimensions.

2 Brane gas dynamics

In this section, we extend the formalism of brane gas dynamics in [16] for branes of arbitrary dimensionality and set up some preliminaries for our calculation. The model we will consider is type II string theory compactified on T^9 . Let us consider the late stage of BV scenario where the radii and curvature scales are grown larger than the ten-dimensional Planck length. When the radii are grown enough, we can neglect the brane-antibrane annihilation. And the excitations on the branes will be red-shifted away faster than the brane tension. Supergravity is a good approximation with the growing radii and falling temperature. Matter fields can be classified into two types. One is the massless supergravity particles corresponding to bosonic and fermionic degrees of freedom. We ignore massive modes since these will decay quickly. The other is wrapped D-branes. These branes do not interact with each other when the radii are grown large. We also ignore the fluctuations within the brane. We assume that the supergravity particles and brane gases are homogeneous for simplicity of calculation. Then we can take the averages of the contributions of all kinds of particles and branes to the energy-momentum tensor.

In the point of string theory, the gravitational interaction is described by the coupled system of the metric and dilaton. Dilaton plays an important role in the large-small symmetry of string theory called T-duality. To be specific, let us start from the following effective action of type II string theory

$$S = \int d^{10}x \sqrt{-g^S} e^{-2\phi} \left[R + 4(\nabla\phi)^2 \right] + S_m, \quad (1)$$

where ϕ is the dilaton field and S_m denote all matter actions including the brane action. Since we are interested in the late time cosmology, we work in the Einstein frame, defined by

$$g_{MN}^S = e^{\frac{1}{2}\phi} g_{MN}^E. \quad (2)$$

In terms of the Einstein metric, the action can be written as

$$S = \int d^{10}x \sqrt{-g^E} \left[R - \frac{1}{2}(\nabla\phi)^2 \right] + S_m. \quad (3)$$

We drop the superscript E from now on. The equations of motion from this action are

$$R_{MN} - \frac{1}{2}g_{MN}R = \frac{1}{2}\nabla_M\phi\nabla_N\phi - \frac{1}{4}g_{MN}(\nabla\phi)^2 - \frac{1}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{MN}}, \quad (4)$$

$$\nabla^2\phi = -\frac{1}{\sqrt{-g}}\frac{\delta S_m}{\delta\phi}. \quad (5)$$

Since the string coupling ($g \equiv e^\phi$) is considered to be small in our assumption of large radii, we do not consider the running of the dilaton throughout this paper. Then the equations of motion are described simply by the Einstein equation

$$R_{MN} - \frac{1}{2}g_{MN}R = -\frac{1}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{MN}}. \quad (6)$$

With the metric ansatz of torus, with $D = 9$,

$$ds^2 = -dt^2 + \sum_{i=1}^D (a_i(t))^2 d\theta_i^2, \quad 0 \leq \theta_i \leq 2\pi, \quad (7)$$

the non-vanishing components of the Einstein tensor are

$$G^t_t = \frac{1}{2} \sum_{k \neq l} \frac{\dot{a}_k \dot{a}_l}{a_k a_l}, \quad (8)$$

$$G^i_i = \sum_{k \neq i} \frac{\ddot{a}_k}{a_k} + \frac{1}{2} \sum_{k \neq l} \frac{\dot{a}_k \dot{a}_l}{a_k a_l} - \sum_{k \neq i} \frac{\dot{a}_k \dot{a}_i}{a_k a_i}, \quad (9)$$

where i is not summed in the second equation.

For the matter part, we first consider a gas of massless supergravity particles, with energy density ρ_S and pressure p_S . Since we assume the gas to be homogeneous and isotropic, we take a perfect fluid form of energy momentum tensor

$$T^M_N = \text{diag}(-\rho_S, p_S, \dots, p_S). \quad (10)$$

The equation of state appropriate for D spatial dimensions fixes $p_S = (1/D)\rho_S$.

The second source of energy momentum comes from a gas of branes, wrapped on the various cycles of the torus. The matter contribution of a single p -brane to the action, in string frame, is represented by the Dirac-Born-Infeld (DBI) action

$$S_p = -T_p \int d^{p+1}\xi e^{-\phi} \sqrt{-\det(g_{\mu\nu} + B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}, \quad (11)$$

where T_p is the tension of p -brane and $g_{\mu\nu}$ is the induced metric to the brane

$$g_{\mu\nu} = g_{MN} \frac{\partial X^M}{\partial \xi^\mu} \frac{\partial X^N}{\partial \xi^\nu}. \quad (12)$$

Here M, N are the indices of $(D+1)$ dimensional bulk spacetime and μ, ν are those of brane. $B_{\mu\nu}$ is the induced antisymmetric tensor field and $F_{\mu\nu}$ is the field strength tensor of gauge fields A_μ living on the brane. With the assumption of grown radii, we neglect $B_{\mu\nu}$ and $F_{\mu\nu}$ terms below. Ignoring the running of the dilaton, we can absorb the effect of constant dilaton into the redefinition of brane tension in the Einstein frame.

Let us consider a gas of 1-branes. The contribution of these branes are characterized by wrapping numbers N_i and \bar{N}_i , where we take N_i to represent the number of 1-branes wrapped on the i cycle, while \bar{N}_i represents the number of anti-1-branes. By symmetry it is a reasonable assumption that they are equal. The wrapping numbers are based on the understanding of thermal fluctuations in the early universe. A single 1-brane action is described by

$$S_{1\text{-brane}} = -T_1 \int d^2\xi \sqrt{-\det g_{\alpha\beta}}, \quad (13)$$

where T_1 is the 1-brane tension. The stress tensor from this 1-brane action is

$$T^{MN} = -T_1 \int d^2\xi \delta^{D+1}(X - X(\xi)) \sqrt{-\det g} g^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^N. \quad (14)$$

As an example, for a single 1-brane wrapped on the 1 cycle and uniformly smeared over the transverse T^{D-1} , the stress energy tensor is

$$T^M_N = -\frac{T_1}{2\pi a_2 \cdots 2\pi a_D} \text{diag}(1, 1, 0, 0, \dots, 0). \quad (15)$$

With wrapping numbers N_i and \bar{N}_i for i cycle, the non-zero components of the 1-brane gas stress tensor are

$$T^t_t = -\frac{T_1}{V} 2\pi \sum_k a_k (N_k + \bar{N}_k), \quad (16)$$

$$T^i_i = -\frac{T_1}{V} 2\pi a_i (N_i + \bar{N}_i). \quad (17)$$

Now we insert these two sources of energy-momentum into the right hand side of the Einstein equations,

$$G^M_N = -8\pi G T^M_N. \quad (18)$$

The time component equation can be solved for the energy density of the supergravity gas,

$$8\pi G \rho_S = \frac{1}{2} \sum_{k \neq l} \frac{\dot{a}_k \dot{a}_l}{a_k a_l} - \frac{8\pi G T_1}{V} (2\pi) \sum_k a_k (N_k + \bar{N}_k). \quad (19)$$

The space component equations, after some algebra, can be reduced to the following set of second-order differential equations

$$\begin{aligned} \frac{\ddot{a}_i}{a_i} = & \frac{8\pi G T_1}{V} \left[\frac{D+1}{D(D-1)} 2\pi \sum_k a_k (N_k + \bar{N}_k) - 2\pi a_i (N_i + \bar{N}_i) \right] \\ & + \frac{1}{2D} \sum_{k \neq l} \frac{\dot{a}_k \dot{a}_l}{a_k a_l} - \sum_{k \neq i} \frac{\dot{a}_k \dot{a}_i}{a_k a_i}. \end{aligned} \quad (20)$$

For a gas of 2-branes, the wrapping can be characterized by N_{ij} , where we take $N_{i < j}$ to represent the number of 2-branes wrapped on the (ij) cycle, while $N_{i > j}$ represents the number of anti-2-branes. The stress energy tensor from a single 2-brane action is

$$T^{MN} = -T_2 \int d^3 \xi \delta^{D+1}(X - X(\xi)) \sqrt{-\det g} g^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^N, \quad (21)$$

where T_2 is the 2-brane tension. Then, for a single 2-brane wrapped on the (12) cycle and uniformly smeared over the transverse T^{D-2} , the stress energy tensor is

$$T^M_N = -\frac{T_2}{2\pi a_3 \cdots 2\pi a_D} \text{diag}(1, 1, 1, 0, \dots, 0). \quad (22)$$

With the wrapping numbers, the non-zero components of stress energy tensor from the 2-brane gas are

$$T^t_t = -\frac{T_2}{V}(2\pi)^2 \sum_{k \neq l} a_k a_l N_{kl}, \quad (23)$$

$$T^i_i = -\frac{T_2}{V}(2\pi)^2 \sum_{k \neq i} a_k a_i (N_{ki} + N_{ik}). \quad (24)$$

Repeating the same procedure as in 1-brane case, we have

$$8\pi G \rho_S = \frac{1}{2} \sum_{k \neq l} \frac{\dot{a}_k \dot{a}_l}{a_k a_l} - \frac{8\pi G T_2}{V} (2\pi)^2 \sum_{k \neq l} a_k a_l N_{kl}, \quad (25)$$

$$\begin{aligned} \frac{\ddot{a}_i}{a_i} &= \frac{8\pi G T_2}{V} \left[\frac{2D+1}{D(D-1)} (2\pi)^2 \sum_{k \neq l} a_k a_l N_{kl} - (2\pi)^2 \sum_{k \neq i} a_k a_i (N_{ki} + N_{ik}) \right] \\ &+ \frac{1}{2D} \sum_{k \neq l} \frac{\dot{a}_k \dot{a}_l}{a_k a_l} - \sum_{k \neq i} \frac{\dot{a}_k \dot{a}_i}{a_k a_i}. \end{aligned} \quad (26)$$

For a gas of 3-branes, we characterize the wrapping numbers of (ijk) cycle as N_{ijk} , where we take N_{ijk} with $\epsilon_{ijk} = 1$ to represent the number of 3-branes wrapped on the (ijk) cycle, while N_{ijk} with $\epsilon_{ijk} = -1$ represents the number of anti-3-branes. The stress energy tensor for a single 3-brane action is

$$T^{MN} = -T_3 \int d^4 \xi \delta^{D+1}(X - X(\xi)) \sqrt{-\det g} g^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^N, \quad (27)$$

where T_3 is the 3-brane tension. For a single 3-brane wrapped on the (123) cycle and uniformly smeared over the transverse T^{D-3} , the stress energy tensor is

$$T^M_N = -\frac{T_3}{2\pi a_4 \cdots 2\pi a_D} \text{diag}(1, 1, 1, 1, 0, \dots, 0). \quad (28)$$

The non-zero components of the 3-brane gas stress energy tensor are

$$T^t_t = -\frac{T_2}{V}(2\pi)^3 \sum_{k \neq l \neq m} \frac{1}{3} a_k a_l a_m N_{klm}, \quad (29)$$

$$T^i_i = -\frac{T_2}{V}(2\pi)^3 \sum_{k \neq l \neq i} \frac{1}{3} a_k a_l a_i (N_{ikl} + N_{lik} + N_{kli}), \quad (30)$$

where $\sum_{k \neq l \neq m}$ means that the sum is taken when all three indices are different. The constraint equation and the second-order differential equations for the radii are

$$8\pi G \rho_S = \frac{1}{2} \sum_{k \neq l} \frac{\dot{a}_k \dot{a}_l}{a_k a_l} - \frac{8\pi G T_3}{V} (2\pi)^3 \sum_{k \neq l \neq m} \frac{1}{3} a_k a_l a_m N_{klm}, \quad (31)$$

$$\begin{aligned} \frac{\ddot{a}_i}{a_i} = & \frac{8\pi G T_3}{V} \left[\frac{3D+1}{D(D-1)} (2\pi)^3 \sum_{k \neq l \neq m} \frac{1}{3} a_k a_l a_m N_{klm} \right. \\ & \left. - (2\pi)^3 \sum_{k \neq l \neq i} \frac{1}{3} a_k a_l a_i (N_{ikl} + N_{lik} + N_{kli}) \right] \\ & + \frac{1}{2D} \sum_{k \neq l} \frac{\dot{a}_k \dot{a}_l}{a_k a_l} - \sum_{k \neq i} \frac{\dot{a}_k \dot{a}_i}{a_k a_i}. \end{aligned} \quad (32)$$

3 Late time behavior

To study the solutions of the field equations (20), (26) and (32), we introduce new variables $\lambda_i(t) \equiv \ln(2\pi a_i(t))$ as in [6]. The field equations when the matter fields are coming from supergravity particles and wrapped 1-brane gas become, from (20),

$$\ddot{\lambda}_i + \frac{\dot{V}}{V} \dot{\lambda}_i = 8\pi G \left(\frac{1}{D} \rho_S + \frac{2}{D-1} \rho_B \right) - \frac{8\pi G T_1}{V} e^{\lambda_i} (N_i + \bar{N}_i), \quad (33)$$

where the volume of the spatial torus V can be expressed as $V = e^{\sum_i \lambda_i}$ and ρ_B is the energy density from 1-brane tension

$$\rho_B = \frac{T_1}{V} \sum_k e^{\lambda_k} (N_k + \bar{N}_k). \quad (34)$$

The energy density from supergravity particles is fixed by the constraint equation (19)

$$8\pi G\rho_S = \frac{1}{2} \sum_{i \neq j} \dot{\lambda}_i \dot{\lambda}_j - 8\pi G\rho_B. \quad (35)$$

This system of equations can be regarded as a non-relativistic particle moving in D dimensions. The particle has a coefficient of friction, given by \dot{V}/V , due to the expansion of the universe. One can consider the right hand side of (33) gives two position-dependent forces acting on the particle [16]. The first term from the supergravity particles,

$$F_i^{(1)} = 8\pi G \left(\frac{1}{D} \rho_S + \frac{2}{D-1} \rho_B \right), \quad (36)$$

is positive definite. This force is common to every value of i and drives a uniform expansion of the universe. The second term from the brane gas contribution,

$$F_i^{(2)} = -\frac{8\pi G T_1}{V} e^{\lambda_i} (N_i + \bar{N}_i), \quad (37)$$

can be either zero or negative depending on the wrapping number. This term can suppress the growth of dimensions when wrapping numbers are nonzero. Thus anisotropic wrapping numbers can lead to an anisotropic expansion of the dimensions.

The constraint equation (35) can be rewritten as

$$\left(\frac{\dot{V}}{V} \right)^2 = \left(\sum_i \dot{\lambda}_i \right)^2 = \sum_i (\dot{\lambda}_i)^2 + 16\pi G (\rho_S + \rho_B). \quad (38)$$

The right hand side is positive definite for all nontrivial cases and we choose the direction of time to make $\dot{V} > 0$.

We are interested in cases where the wrapping number N_i is anisotropic. We can classify the spatial dimensions into two kinds. We refer to a direction i as *unwrapped* if $N_i = \bar{N}_i = 0$ and as *wrapped* if $N_i = \bar{N}_i \neq 0$. Now consider the case when m dimensions are unwrapped and $D - m$ dimensions are wrapped. This configuration can be achieved by thermal fluctuations in the early universe through the mechanism of Brandenberger and Vafa [5]. Denote the two subvolumes as in [16]

$$\mu = \sum_{i=1}^m \lambda_i = \ln (\text{volume of unwrapped torus}), \quad (39)$$

$$\Lambda = \sum_{i=m+1}^D \lambda_i = \ln (\text{volume of wrapped torus}). \quad (40)$$

Summing over the appropriate values of i , and using the definition of ρ_B , we find the following differential equations for μ and Λ

$$\ddot{\mu} + (\dot{\mu} + \dot{\Lambda})\dot{\mu} = 8\pi G \left(\frac{m}{D} \rho_S + \frac{2m}{D-1} \rho_B \right), \quad (41)$$

$$\ddot{\Lambda} + (\dot{\mu} + \dot{\Lambda})\dot{\Lambda} = 8\pi G \left(\frac{D-m}{D} \rho_S + \frac{D-2m+1}{D-1} \rho_B \right). \quad (42)$$

Repeating the same procedure when the matter fields are coming from supergravity particles and wrapped 2-brane gas, we have, from (26),

$$\ddot{\lambda}_i + \frac{\dot{V}}{V} \dot{\lambda}_i = 8\pi G \left(\frac{1}{D} \rho_S + \frac{3}{D-1} \rho_B \right) - \frac{8\pi G T_2}{V} e^{\lambda_i} \sum_{j \neq i} (N_{ij} + N_{ji}) e^{\lambda_j}. \quad (43)$$

The constraint equation has the same form as (35) with the energy density ρ_B replaced by

$$\rho_B = \frac{T_2}{V} \sum_{i \neq j} e^{\lambda_i} e^{\lambda_j} N_{ij}. \quad (44)$$

In this case we can classify the spatial dimensions into three classes [16]. A direction i is referred to as *unwrapped* if $N_{ij} = N_{ji} = 0$ for all j . Directions i for which N_{ij} and N_{ji} are nonzero except for those j corresponding to an unwrapped direction are referred to as *fully wrapped*. Directions i where some of the N_{ij} or N_{ji} are zero for values of j which are not unwrapped are referred to as *partially wrapped*. The equations governing the motion of μ and Λ are

$$\ddot{\mu} + (\dot{\mu} + \dot{\Lambda})\dot{\mu} = 8\pi G \left(\frac{m}{D} \rho_S + \frac{3m}{D-1} \rho_B \right), \quad (45)$$

$$\ddot{\Lambda} + (\dot{\mu} + \dot{\Lambda})\dot{\Lambda} = 8\pi G \left(\frac{D-m}{D} \rho_S + \frac{D-3m+2}{D-1} \rho_B \right). \quad (46)$$

Similarly when the matter fields are coming from supergravity particles and wrapped 3-brane gas, we have, from (32),

$$\begin{aligned}\ddot{\lambda}_i + \frac{\dot{V}}{V}\dot{\lambda}_i &= 8\pi G \left(\frac{1}{D}\rho_S + \frac{4}{D-1}\rho_B \right) \\ &- \frac{8\pi GT_3}{V}e^{\lambda_i} \sum_{j \neq k \neq i} \frac{1}{3}e^{\lambda_j}e^{\lambda_k}(N_{ijk} + N_{kij} + N_{jki}).\end{aligned}\quad (47)$$

The constraint equation has the same form as (35) with the brane energy density ρ_B replaced by

$$\rho_B = \frac{T_3}{V} \sum_{i \neq j \neq k} \frac{1}{3}e^{\lambda_i}e^{\lambda_j}e^{\lambda_k}N_{ijk}.\quad (48)$$

Also we can classify the spatial dimensions into three classes as in 2-brane case; *unwrapped*, *partially wrapped* and *fully wrapped*. The equations for μ and Λ are

$$\ddot{\mu} + (\dot{\mu} + \dot{\Lambda})\dot{\mu} = 8\pi G \left(\frac{m}{D}\rho_S + \frac{4m}{D-1}\rho_B \right),\quad (49)$$

$$\ddot{\Lambda} + (\dot{\mu} + \dot{\Lambda})\dot{\Lambda} = 8\pi G \left(\frac{D-m}{D}\rho_S + \frac{D-4m+3}{D-1}\rho_B \right).\quad (50)$$

Generalizing the above result to the case when matter fields are coming from supergravity particles and wrapped p -brane gas, one can find

$$\ddot{\mu} + (\dot{\mu} + \dot{\Lambda})\dot{\mu} = 8\pi G \left(\frac{m}{D}\rho_S + \frac{(p+1)m}{D-1}\rho_B \right),\quad (51)$$

$$\ddot{\Lambda} + (\dot{\mu} + \dot{\Lambda})\dot{\Lambda} = 8\pi G \left(\frac{D-m}{D}\rho_S + \frac{D-(p+1)m+p}{D-1}\rho_B \right).\quad (52)$$

The evolution of unwrapped subvolume μ and wrapped subvolume Λ depends on the number of unwrapped dimensionality m . For small m , both terms on the right hand side of $\ddot{\Lambda}$ equation (52) are positive. If $\dot{\Lambda}$ is zero, then the second derivative must be positive, leading to a local minimum. Conversely, for a larger value of m , the right hand side of this equation has both a positive and a negative term. In this case both local maxima and minima

are possible. However, for unwrapped subvolume $\ddot{\mu}$, there are only positive terms on the right hand side. So if these directions are initially expanding they will expand forever.

When the second term of right hand side in $\ddot{\Lambda}$ equation vanishes the brane tension does not contribute to the growth of the internal dimensions. This gives a criterion for the critical spatial dimensionality of the unwrapped subspace

$$\begin{aligned} D - 2m_c + 1 &= 0, & \text{for } 1\text{-brane}, \\ D - 3m_c + 2 &= 0, & \text{for } 2\text{-brane}, \\ D - 4m_c + 3 &= 0, & \text{for } 3\text{-brane}. \end{aligned} \tag{53}$$

For $D = 9$, this occurs at $m_c = 5, 11/3$ and 3 for 1-, 2- and 3-brane gases respectively. From this result we can conclude that wrapping the internal dimensions with 1- and 2-branes cannot yield the observed three large spatial dimensions. For 3-brane there is no contribution from ρ_B but Λ will grow by ρ_S . However the growing rate is smaller than that of μ . When μ is wrapped by p -dimensional brane gas, the criterion is

$$D - (p + 1)m_c + p = 0, \quad \text{for } p\text{-brane}. \tag{54}$$

As an example, for $p = 4$, this gives $m_c = 13/5$. This means that when $m > 13/5$ there can be negative contribution from the ρ_B terms. Our analysis implies that the compact extra dimensions should be wrapped with three or higher dimensional branes for the observed three large spatial dimensions. If they are wrapped by just one or two dimensional branes, they cannot remain compact.

4 Conclusion and Discussion

We have studied the late-time behavior of the universe in the framework of brane gas cosmology. For the cases when different dimensions are anisotropically wrapped by various branes, we have derived Friedman-like equations governing the dynamics of wrapped and unwrapped subvolumes. We pointed out that one cannot keep the extra dimensions compact with one or two dimensional brane gases.

Though the wrapped directions grow slowly compared with the unwrapped dimensions, it is not guaranteed that the compact dimensions can be stabilized. If four or higher dimensional branes are frozen at the end of thermal

stage, these branes give negative force to Λ while giving positive force to μ . If we assume that every term in ρ_B takes a comparable contribution to the energy density, we can replace the effect of all wrapped branes with a $(D - m)$ -brane gas. In this point of view, one can study the evolution of a universe starting from the metric

$$g_{MN} = \text{diag}(-1, a^2 \delta_{ij}, b^2 \delta_{mn}), \quad (55)$$

where a^2 is the scale factor of the three dimensional space and b is the scale factor of the internal $D - 3$ dimensional subspace. The stabilization of the extra dimensions with this asymmetric setting was attempted in the brane world scenario [18]. Recently the stabilization of extra dimensions was studied numerically. In string gas cosmology, cosmological solution for the late time evolution showed that large dimensions continue to expand and the small ones are kept undetectably small [19]. In [20], it is shown that six compact dimensions become stabilized at the self-dual radius while three dimensions grow large. More recently it has been shown that, by reducing the effect of string gas to the four dimensional Einstein gravity, string modes cannot stabilize the internal dimensions wrapped by winding strings, except in the special case of one extra dimension [21]. However, the evolution of extra dimensions is slow enough compared to the unwrapped large dimensions.

One way to achieve the stability of the internal dimensions is to include the effect of fluxes. Alexander [15] pointed out the possibility of stabilizing the extra dimensions with G-flux in the context of brane gases in eleven dimensional supergravity derived from M-theory. We expect similar argument can be applied to brane gases in ten dimensional supergravity from type II string theory. For example, if we consider the effect of NS-NS sector in the bulk action, the cosmology is described by

$$S = \int d^{10}x \sqrt{-g^S} e^{-2\phi} \left[R + 4(\nabla\phi)^2 - \frac{1}{12} H_{LMN}^2 \right] + S_m, \quad (56)$$

which involves the dynamics of two-form field B_{MN} through $H_{LMN} = 3\nabla_{[L} B_{MN]}$. Then the stability of the internal dimensions can be studied by the coupled three equations of motion involving g_{MN} , ϕ and H_{LMN} instead of two (Eqs. 4 and 5). We expect an effective potential with a confining form so that the internal dimensional will oscillate and remain small.

Classical supergravity holds when all radii are larger than the ten dimensional Planck length. So the equations we used are valid when the radii are either constant or growing with time. In this case we can safely neglect the massless excitations on the branes which will redshift quickly. We can also

neglect the brane antibrane annihilations because branes will be frozen as the transverse dimensions expand. However, in string theory, gravitational interaction is described not by metric alone but by the coupled system of metric and dilaton. The assumption that dilaton is constant may be inconsistent with the cosmological equations describing the evolution. Also the production of stringy objects in the general time dependent background is complicated since the back reaction of such strings or branes is likely to be important. Further studies on the late-time behavior of brane gas cosmology with interacting branes together with dilaton field are needed.

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